Maximizing Continuous Barrier Coverage in Energy Harvesting Sensor Networks

Jonathan DeWitt
Department of Computer Science
Haverford College
Haverford, Pennsylvania 19041
Email: jdewitt@haverford.edu

Suzanne Patt
Department of Math and Computer Science
Ripon College
Ripon, Wisconsin 54971
Email: patts@ripon.edu

Hongchi Shi
Department of Computer Science
Texas State University
San Marcos, Texas 78666
Email: hs15@txstate.edu

***Abstract*—The lifetime of wireless sensor networks (WSNs) has been studied extensively. Lifetime is application specific, and long lifetime is desirable in general. In barrier coverage, it is desirable to have a large number of barriers active as well. Energy harvesting is an active research area in WSNs. In this paper we study the barrier coverage problem in energy harvesting WSNs. We develop an algorithm that finds a repeating sleep/wakeup schedule that can provide the continuous operation of the maximum number of barriers in an energy harvesting WSN.**

I. Introduction

The lifetime of wireless sensor networks (WSNs) has been studied extensively [4]. Long lifetime is desirable. For instance, it can make the deployment of a network along a difficult border much less cost-prohibitive as it would reduce the need for frequent maintenance. However, network lifetime is application specific and thus necessitates individual study for different applications.

One WSN application is detection of intruders for infrastructure protection and border control. For this application, barriers made of sensors are set up to detect intruders' crossing of the barriers. The k-barrier coverage problem is to set up k disjoint barriers in a wireless sensor network with an objective of achieving the longest operation time [12], [13]. The network lifetime for k-barrier coverage is the longest time of having k operational disjoint barriers. The parameter k has a large impact on the quality of the barrier coverage: the larger the k, the more robust the barrier coverage.

Harvesting energy in WSNs has become an active research area [2], [6], [8], [9], [14], [17], [16]. A recent survey on energy harvesting in sensor networks [17] observes that "energy harvesting techniques have the potential to address the trade off between performance parameters and lifetime of sensor nodes." It is obvious by looking at the harvesting and sensing costs [16] that a naive approach to barrier coverage will not work. In a network that gathers energy with vibrational sensors harvesting at most $.1mW$ that also employs light sensors using $1.155mW$, it is not hard to see that such a network cannot be powered indefinitely in a barrier coverage application. Energy harvesting rates are much lower than energy consuming rates in sensor nodes. With energy harvesting techniques, there is still research to be done on extending network lifetime and increasing coverage.

To the best of our knowledge, the k-barrier coverage problem in WSNs with energy harvesting capabilities is still a topic that has yet to be studied. In this paper, we study the k-barrier coverage problem in a framework that allows energy harvesting and present algorithms achieving the maximum continuous barrier coverage from such a network.

II. Related Work

In this paper, we extend the results of Kumar et al. [13] who proposed an optimal solution to achieving k-barrier coverage without energy harvesting. The solution also has applications in achieving coverage in non-disk sensing regions and heterogeneous sensing regions as it is based on a coverage graph. The solution relies on using m-route flows to schedule the nodes such that they achieve an upper bound on the lifetime of the network, thus maximizing the lifetime.

Kishimoto and Takeuchi largely introduced the topic of m-route flows in a network [11], and Kishimoto presented an algorithm for this [10]. Aggarwal and Orlin improved Kishimoto's algorithm by finding a solution with better run time but with the added requirement that all of the capacities of the flow be integers [1]. Du and Chandrasekaran [5] reexamined the m-route maximum flow problem and presented two new algorithms.

Energy harvesting has been studied in tandem with WSNs which can be hard to maintain due to problems with deployment and energy constraints. Raghunathan et al. surveyed several techniques to reduce energy consumption, including energy aware platforms for information processing and communication protocols for sensor collaboration [15]. Jiang et al. studied the use of solar power to extend the life of their networks [8]. Arms et al. demonstrated the ability of wireless sensor nodes to operate at very low power levels while working together with an energy harvesting system and presented methods to dramatically increase the lifetime of the network [2]. Gilbert and Balouchi gave an overview of some of the different types of energy available for harvesting, including electromagnetic radiation, thermal, and vibration [6]. They discussed the applications of each type of energy and the type of hardware associated with the various methods of energy harvesting. They also discussed different methods to harvest energy from the environment, including power from human movement, and presented two different case studies. The two scenarios show that energy harvesting has a wide range of applications in different environments. It can be difficult to find an appropriate schedule for WSNs with energy harvesting capabilities due to the changing environmental conditions. Kansal and Srivastava presented a solution to this problem [9], and Vigorito et al. extended their results [18]. Their solutions allow a sensor network to make the most of the energy available in the environment around the network.

III. Model

In this section, we present the model for the WSNs we are considering.

Definition 1. The coverage graph of a network, N, is the graph $G(N) = (V, E)$ such that V is a set of nodes in one-to-one correspondence with the sensors of N in addition to two virtual nodes at the left and right boundaries of the sensing region, s and t. Two nodes v_i and v_j are connected by an edge $(v_i, v_j) \in E$ if the sensing regions of the corresponding sensors overlap [12].

We can now define the notion of barrier coverage.

Definition 2. A network N provides k-barrier coverage if and only if there exist k node-disjoint paths between the two virtual nodes s and t in $G(N)$ [12].

Definition 3. A homogeneous harvesting coverage graph, $G(N)$, of a sensor network, N is a coverage graph with a harvesting function $B(v)$ such that each node harvests energy at some fixed rate, β, a sensing cost function, $A(v)$, such that each node expends at a fixed rate, α, as well as a battery capacity function, $U(v) = \epsilon$, for all $v \in V$.

We also examine the heterogeneous case, in which the rates vary by node. This more realistically reflects the uncertainty of applications.

Definition 4. A heterogeneous harvesting coverage graph, $G_h(V, E)$, is a coverage graph $G(V, E)$ with a harvesting function, $B(v_i) = \beta_i$, energy use function $A(v_i) = \alpha_i$, and a battery capacity function, $U(v_i) = \epsilon_i$, for all $v_i \in V$.

Now that we have defined the networks, we present our framework.

A. Node Quality of Service

Throughout the rest of the paper, it will be inconvenient to refer to α_i and β_i individually. Instead we refer to $q(v_i)$, the proportion of the time that v_i can remain on.

We can justify this model by noting that many energy harvesting techniques harvest energy at a constant rate. Gilbert and Balouchi showed several methods of energy harvesting like this such as harvesting of steady state mechanical energy, vibrational energy, or electromagnetic radiation [6]. The simplicity of this model makes the analysis tractable while also not requiring algorithms that are too complex for relatively simple sensors to run.

Wang et al. [19] discuss barrier coverage in wireless sensor networks with adjustable sensing ranges. This models a network in which each node consumes power at some constant rate, yet different nodes have different energy expenditure rates. Similarly, when we consider energy harvesting, we should not assume that each node is able to harvest at the same rate. For instance, if we consider a network deployed in a vibrating environment [6], we should not assume that every node is vibrating at the same amplitude; and similarly in an radio energy harvesting scenario, distances from the source can affect the energy harvested, and thus we should not assume that all the rates are constant [20]. In this way, the nodes in the wireless sensor network replenish their energy supply at unique yet fairly fixed rates.

Definition 5. The quality of service, $q(v)$, for a node $v \in V$ is the maximum proportion of the time that the node is able to remain on in a repeating sleep/wake schedule.

We now derive $q(v)$ for two cases.

Lemma III.1. Given a node v with $A(v) = \alpha$ and $B(v) = \beta$ that charges continuously, its quality of service, $q(v)$, is $\frac{\beta}{\alpha}$. If it only charges while it is not providing coverage then $q(v) = \frac{\beta}{\alpha+\beta}$.

Proof. In the continuous charging case, first note that T_{on} is the maximum amount of time the node stays on during the T_{total} time period. Since the expended energy cannot be greater than the harvested energy for the continuous operation of a sensor, $T_{on}\alpha = T_{total}\beta$. Thus, in the continuous charging case, we find:

$$q(v) = \frac{T_{on}}{T_{total}} = \frac{\beta}{\alpha} \tag{1}$$

In the non-continuous charging case, note that the node only charges while it is not on, so we have $T_{on}\alpha = (T_{total} - T_{on})\beta$, which means $T_{on}(\alpha + \beta) = T_{total}\beta$. Thus, we have for the non-continuous charging case:

$$q(v) = \frac{T_{on}}{T_{total}} = \frac{\beta}{\alpha + \beta} \tag{2}$$

□

B. Lifetime and Maximum Continuous Coverage

Lifetime and maximum coverage are the most important metrics for the quality of barrier coverage.

Definition 6. If a sensor network, N, provides $K(t)$-barrier coverage at time t, then the lifetime of the k-barrier coverage is the maximum t such that $K(t) \geq k$ [4].

In the case that barriers are able to provide coverage indefinitely due to their energy harvesting capability, the lifetime metric becomes meaningless. As the parameter k in the k-barrier coverage measures the robustness of the barrier coverage, we propose the *maximum continuous barrier coverage* as the metric of barrier coverage quality.

Definition 7. If a sensor network, N, provides $K(t)$-barrier coverage at time $t \in (0, \infty)$, then the *maximum continuous barrier coverage*, $\kappa(N)$, is $\min\{K(t) \mid t \in (0, \infty)\}$.

We assume that $q(v_i) \in (0, 1)$ to make this problem non-trivial, as otherwise, all the nodes could simply run all the time. Because of the limited battery capacity, however, scheduling is also a concern. We show that there exists a repeating schedule in which these proportions, $q(v_i)$, can always be achieved.

Definition 8. A schedule S for a senor node is *unfeasible* if at some point $t \in [0, \infty)$ the remaining energy, $u(t)$, of a node is negative. Otherwise, a schedule is *feasible*.

Lemma III.2. Given a fully charged node, v, if a repeating unit schedule has $T_{on} \leq q(v)$ and $T_{off} \geq 1 - q(v)$, then it is feasible and can be repeated indefinitely.

Proof. We will prove this using proof by contrapositive. To do this we show that if a schedule is unfeasible, then $T_{on} > q(v)$ and $T_{off} < 1 - q(v)$. Let $u(t)$ be the energy remaining in a node at time t. As the schedule is unfeasible, $u(t) < 0$ for some time $t > 0$. We pick this minimum t such that $u(t) < 0$. Now we examine the unit interval $[t-1, t]$. At the beginning of the interval $u(t-1) > 0$, because t is minimal. Let T_{on} be the amount of time during the interval that the sensor senses, and T_c is the total time that the sensor is charging. Here we examine only the continuously charging case. This implies that $u(t-1) - \alpha T_{on} + \beta T_c < 0$ because $u(t) < 0$. Since $u(t-1) > 0$, we can drop that term to find: $\beta T_c < \alpha T_{on}$, which implies $\frac{\beta}{\alpha} < \frac{T_{on}}{T_c} = T_{on}$ because $T_c = 1$ as we are examining a unit interval. Thus $\frac{\beta}{\alpha} = q(v) < T_{on}$, completing the proof. □

It is essential to note here that the battery capacity of v, $U(v)$, appears nowhere in the above formulae. Later, we will see that this implies that battery capacity does not affect maximum continuous barrier coverage of the network if we schedule the nodes correctly.

IV. Homogeneous Case

In this section, we derive an upper bound on the maximum continuous barrier coverage that a homogeneous sensor network, N, can provide, $\kappa(N)$. Then we present an algorithm that schedules the nodes to achieve the bound, thus maximizing $\kappa(N)$.

A. Upper Bound

Consider a homogeneous sensor network N. We can now examine $\kappa(N)$ purely in terms of $q(v)$ and thus keep our results sufficiently general.

Lemma IV.1. Consider a homogeneous sensor network N and suppose $l \geq k$ is the maximum number of node-disjoint paths between the two virtual nodes s and t in N. Then the maximum continuous barrier coverage that the network can provide is $\kappa(N) = \lfloor lq(v) \rfloor$.

Proof. From our assumption, we know that there are at most l node-disjoint paths from s to t in N. By Menger's Theorem [3], we know that there is a set D of d nodes that when removed will disconnect N. We call these nodes in D critical. From Definition 2, we know that for the network to provide k-barrier coverage, k node-disjoint paths must be active at any time. Consider a unit time interval, $T_0 = [0, 1]$, during which a sensor v in a path can run for $q(v)$ time. At any point at least k of the critical nodes must be active, so the maximum amount of time that they can remain active during the interval is $\frac{q(v)l}{k}$. We pick the maximum $k \in \mathbb{Z}_+$ such that $\frac{q(v)l}{k} \geq 1$. Now, we multiply through by k, and in order to ensure we get an integral number of barriers, we apply the floor function. Thus we have $k = \lfloor lq(v) \rfloor = \kappa(N)$. □

It is important to note here that the upper bound does not rely on the battery capacity of any of the nodes, rather it relies on their ability to remain turned on for a fixed proportion of every time interval.

Using the above lemma, we can solve the equation $\kappa(N) = \lfloor lq(v) \rfloor$ backward by substituting α and β back in order to calculate what those values would have to be in order to achieve a certain level of coverage for a specific network.

B. Reaching the Upper Bound

We now present an algorithm that achieves the upper bound. To fully understand the benefits of the algorithm, we give a brief discussion of path switches.

1) Path Switches: In a network, it is desirable to minimize the number of times that a node is turned on and off. In the case of limited lifetime, path switches are undesirable.

Definition 9. A *Path Switch* occurs when a group of sensors that together provide 1-barrier coverage is turned off and is later turned on as a group. If this group of sensors exhausts its lifetime once it is turned on, then this group of sensors has no path switches [13].

We extend the definition of a path switch by using the battery capacity to judge when a switch is unneeded for the case when lifetime is unlimited.

Definition 10. In a homogeneous sensor network, N, a *continuous path switch* occurs when a path is turned on, but not allowed to run for its entire battery life, which in this case is $\frac{\epsilon_0}{\alpha - \beta}$ units of time for the continuous charging case and $\frac{\epsilon_0}{\alpha}$ for the non-continuous charging case, where ϵ_0 is the initial energy.

The algorithm presented here uses no continuous path switches in achieving the maximum continuous barrier coverage.

2) Homogeneous Algorithm: The algorithm functions by balancing the running times of the barriers so that k barriers are always on at a time and no barrier ever has to run for less than its entire lifetime. To do this, the algorithm loops through a fixed interval setting every barrier to be on for the maximum amount of time it can be on, and when coming to a next barrier it turns that barrier on when the last one is turned off. This schedule with t_w for working and t_s for sleeping can then be continuously repeated, ensuring $\kappa(N)$-barrier coverage.

We now prove that the above algorithm provides the maximum continuous barrier coverage for a network in the case that continuous coverage can be provided.

Theorem IV.2. Given a network, N, with battery capacity, ϵ, energy depletion rate, α, and recharge rate, β, the schedule provided by the homogeneous algorithm achieves $\kappa(N)$-barrier coverage.

input : A Sensor Network N, with battery capacity ϵ, energy depletion rate α, and recharge rate β.
output: A schedule $(t_w(v_i), t_s(v_i))$ for all v_i that achieves $\kappa(N)$-barrier coverage.

Compute the coverage graph, $G(N)$;
Compute the maximal set of node-disjoint paths, L, through $G(N)$ between the two virtual nodes, s and t;
$C = \frac{\epsilon}{\alpha-\beta} + \frac{\epsilon}{\beta}$;
$T_{on} = \frac{\epsilon}{\alpha-\beta}$;
$T \leftarrow 0$;
for $p \in L$ **do**
    $T' \leftarrow (T + C)$;
    if $T' \geq C$ **then**
        $T' \leftarrow T' - C$;
    end
    $\forall v \in p - \{s,t\}, t_w(v) \leftarrow T$ and $t_s(v) \leftarrow T'$;
    $T \leftarrow T'$;
end

Algorithm 1: The homogeneous case

Proof. We show that at any point during the schedule created by the algorithm there are at least l barriers active. Then, we show that $l \geq \kappa(N)$. We prove the algorithm for the unit time interval and $q(v)$ as this is general. The algorithm as written above gives the answer for the continuous case which implies that C, the length of the repeating schedule is $\frac{\epsilon}{\alpha-\beta} + \frac{\epsilon}{\beta}$. That number can be derived simply by starting with a node at 0 charge, charging it and then adding the charging time to the depletion time, which can be derived from the formula $\epsilon + T\beta - T\alpha \geq 0$.

Suppose that $G(N)$ has a maximum of l disjoint barriers. The algorithm schedules the coverage for each interval by setting $t_w(v_{i+1}) = t_s(v_i)$ and then looping back around to the beginning of the interval when it encounters a $t_s(v_i) > C$. From this we can see that if each barrier is able to remain on for $q(v)$ portion of the interval, then it will require $\lfloor \frac{1}{q(v)} \rfloor$ of the l barriers to provide 1-barrier coverage continuously. Similarly it requires $\lfloor \frac{k}{q(v)} \rfloor$ barriers to provide k-barrier coverage continuously. The algorithm finishes scheduling for providing 1-barrier coverage before it begins to schedule for providing 2-barrier coverage. Thus when the algorithm schedules $\lfloor \frac{k}{q(v)} \rfloor = l$ paths it creates k barriers. This implies that $k = \lfloor lq(v) \rfloor$ barriers, which is the upper bound that was derived before.

Further, we assume that all of the nodes start at full charge and thus do not need to have charged throughout the first cycle. In subsequent cycles, all of the nodes are sufficiently charged to run because they have been asleep for $1 - q(v)$ units of time at the point they are woken. If we want to use this algorithm to schedule nodes that charge only in the off state, then we set C and T_{on} as follows: $C = \frac{\epsilon}{\alpha} + \frac{\epsilon}{\beta}$ and $T_{on} = \frac{\epsilon}{\alpha}$. ☐

While the above algorithm is fairly straight forward, it is better than repurposing the *Stint* algorithm [13] to balance energy usage, even though that algorithm would also manage to achieve the desired maximum level of coverage. The advantage of the algorithm proposed here is that it does not cause any continuous path switches to occur, whereas Kumar's algorithm [13] does use path switches.

Lemma IV.3. The homogeneous algorithm uses no continuous path switches.

Proof. Note that the difference between $t_w(v)$ and $t_s(v)$ is equal to $q(v)$ in the unit case, which is the maximum amount of time that the nodes can provide service in a unit interval. Thus the nodes turn off when they have exhausted their battery life and turn on only when they have finished charging. ☐

Complexity: The complexity of this algorithm is clearly dominated by the requirement of calculating the disjoint paths in the coverage graph, $G(N)$. One algorithm that does this in a distributed manner is He and Shi's modification [7] of the Goldberg-Tarjan algorithm. In typical sensor network applications where all edges have unit cost, their PUSH-PULL-IMPROVE algorithm has a time complexity of $O(|V|^2 \log |V|)$ and a message complexity of $O(|V|^2 \log |V|)$.

V. Heterogeneous Case

We now relax the assumption that the nodes in the network are identical. We will derive a new upper bound for the maximum continuous coverage the network can provide and present an algorithm that achieves the bound.

A. Upper Bound

When we relax the assumption that the nodes are identical, we must change the framework slightly in order to still talk in the same terms. In this new case, we consider a network, N, with an associated coverage graph, $G(V, E)$, and functions mapping the nodes $v_i \in V$ to corresponding levels of energy consumption, α_i, energy harvesting, β_i, and battery capacity ϵ_i. From these new values, we can derive a new $q(v_i) = q_i$, which, as before, is the maximum proportion of an interval that v_i is able to sense in a repeating schedule. We retain the assumption that $q(v_i) \in (0, 1)$, as otherwise the problem would be trivial to solve.

The problem then more generally becomes finding the maximal schedule in the case that each node may have a unique $q(v_i)$. To analyze this problem we treat the problem as a max-flow problem, so we need the following definitions:

Definition 11. [13] An s-t $flow$ in $G(N)$ is a mapping $f : E \rightarrow \mathbb{R}^+$ such that

1) $\forall w \in V \backslash \{s,t\}, \sum_{(w,v)\in E} f(w,v) = \sum_{(v,w)\in E} f(v,w)$
2) $\sum_{(s,v)\in E} f(s,v) = \sum_{(v,t)\in E} f(v,t)$ and
3) $\forall w \in V \setminus \{s,t\}, \sum_{(w,v)\in E} f(w,v) \leq c(w)$.

Definition 12. An s-t $path\ flow$ is an s-t flow in $G(N)$ with the property that the flow network is a single path from s to t [13].

Definition 13. An $elementary\ edge\text{-}m\text{-}route$ flow from s to t is a subnetwork of $G(N)$ that is a composition of m s-t path flows such that the flow through each edge is constant. We call this constant the value, a, of the *elementary edge-m-route*, and thus the capacity of such a flow is $a \cdot m$ [11].

Definition 14. An *edge-m-route* flow is a composition of *elementary edge-m-route* flows in $G(N)$ that is an s-t flow. Its capacity is the sum of the capacities of the elementary flows, and its value is $\sum_A a_i$, the sum of the values of the elementary flows [11].

We consider vertex-m-route flows here in which each vertex has a capacity. These are studied as edge-m-route flows in which a vertex is divided into an in and an out vertex connected by an edge of the capacity of the vertex.

We set the capacity of $v \in V$ to $c(v_i) = q(v_i)$, and call this graph $G_q(V, E)$. Now notice that in this graph each elementary edge-m-route flow of value a represents a proportion of the time in which that flow can be used in a unit time cycle of scheduling because the amount of flow through a vertex is never more than the maximum proportion of time that the node can be on. We can now give our result.

Theorem V.1. Given a sensor network, N, the network can achieve continuous k-barrier coverage if and only if there is an s-t k-route flow of value ≥ 1 in $G_q(N)$, which is a k-route flow of capacity $f_k \geq k$.

Proof. First, we show that the ability to provide continuous k-barrier coverage implies that the network has a k-route flow of value greater than 1, which implies that the total flow through the network is at least k. We examine a unit schedule to make the analysis more straightforward. From the assumption, at each time $t \in [0, 1]$ there are at least k barriers active. Let A be the ordered set of all combinations of k barriers used during the schedule. Let $t(a_i)$, where $a_i \in A$, denote the length of time that the ith set of barriers are on. We can identify the ith set of k-barriers with an elementary k-route flow, because each node in a_i is on for the same amount of time, so we can identify $t(a_i)$ with the value of the elementary k-route flow. From the assumption that this schedule is continuous, we must have $\sum t(a_i) \geq 1$, which implies that the value of the composition of these elementary k-route flows is at least 1. Note that this flow is feasible, as the schedule being feasible implies that the flow through each node, v, must be no more than $q(v)$

Now we show that having a k-route flow, F_k, with value $f_k > 1$ implies that we can find a feasible schedule for providing continuous k-barrier coverage. First we divide F_k into elementary k-route flows. As before, we denote the ith elementary k-route flow as a_i, and its value as $t(a_i)$. From the assumption we have $\sum t(a_i) \geq 1$. We can now use each elementary k-route flow as part of a schedule, as each one of them represents a proportion of the time that we can run that elementary k-route flow to provide barrier coverage. Because $\sum t(a_i) \geq 1$, this will cover a unit interval. Because F_k is a flow, the time each node will stay on in this schedule is less than $q(v_i)$, so by Lemma III.2 it is feasible. This completes the proof. □

Now that we have established an upper bound on $\kappa(N)$ in V.1, we will present our algorithm that achieves that bound.

B. Algorithm

input : A Sensor Network, N, with battery capacities ϵ_i, energy depletion rates, α_i, and recharge rates β_i, and the maximum barrier number k
. **output**: A schedule $(t_w(v), t_s(v))$ for all of the nodes that achieves the maximum continuous barrier coverage.

Compute the $q(v_i)$ of each node from α_i and β_i;
Calculate the minimum cycle length, $T_C = \min_{v_i \in V} \epsilon_i(\frac{1}{\alpha_i - \beta_i} + \frac{1}{\beta_i})$;
Compute the coverage graph, $G(N)$;
Compute the m-route max flow through $G(N)$ between the two virtual nodes, s and t;
Decompose that m-route flow into a set of elementary k-route-flows, F;
$T \leftarrow 0$;
for $r \in F$ **do**
    $T' \leftarrow T + value(r) * T_C$;
    $\forall v \in r - \{s, t\}, t_w(v) \leftarrow T$ and $t_s(v) \leftarrow T'$;
    $T \leftarrow T'$;
end

Algorithm 2: The heterogeneous case

We now give a proof that this algorithm always provides the maximum level of coverage given $\kappa(N)$.

Theorem V.2. The *heterogeneous algorithm* creates a sleep/wake schedule for a network N that provides $\kappa(N)$ continuous barrier coverage.

Proof. Consider a sensor network, N, and its coverage graph $G_q(N)$. Let $f_k(G_q)$ denote the value of the largest $\kappa(N)$-route flow through N. To show that the algorithm is able to achieve this maximum level of coverage we must show that it finds that level of coverage when its preconditions are met.

To scale to avoid the problem of limited battery capacity, the algorithm finds the minimum length of the the maximum cycle for some node and then makes that amount of time the unit schedule, as it is feasible for all nodes. This value is equal to $T_C = \min_{v_i \in V} \epsilon_i(\frac{1}{\alpha_i - \beta_i} + \frac{1}{\beta_i})$ in the continuous case and all other values are scaled to it.

The algorithm calculates the value of $f_\kappa(G_q)$ as well as the elementary-κ-route flows composing that κ-route flow. By the usage of κ, we assume that the value of the flow $f_\kappa(G_q) \geq 1$ and thus summing the capacities of each elementary-κ-route flow is $f_\kappa(G_q)$. The algorithm runs each elementary-κ-flow, r, for value$(r)\dot{T}_C$ time providing κ barrier coverage for that period. $f_\kappa(G_q) \geq 1$ this implies that $f_\kappa(G_q)T_C \geq T_C$, and thus covers the interval. By Lemma III.2 this is feasible because the flow is valid as $q(v_i)$ is never exceeded, completing the proof. □

If the the value of the k-flow is less than $\kappa(N)$, then the schedule will have a period when it provides no service. If there are more elementary flows than needed to provide coverage, then the extra flows will run as well, as the algorithm scales the flows down to a schedule where the unit is the minimum feasible cycle. If such a behavior is undesirable in the application, then the algorithm can be instructed to finish after completing the loop in which $T' \geq T_C$ and to set $T' \leftarrow T_C$ for that iteration.

Complexity: The complexity of this algorithm is clearly dominated by the calculation of the elementary m-route flows of the network. In a similar application, Kumar [13] uses Kishimoto's SEM algorithm [10] for finding the flows. Aggarwal and Orlin's algorithm [1], while more efficient in the unit capacity case, requires that the capacity of every edge of the graph be an integer and has a complexity that increases as $\log(C_{max})$ [5]. Because we are dealing with capacities that are not necessarily integral and range from 0 to 1, we would have to scale all of these numbers up in order to use the algorithm. Thus, their algorithm is not ideal for this application as it would require a large C_{max}. Another algorithm we can consider is the Newton's Method algorithm proposed by Du and Chandrasekaran [5]. Their approach uses the concavity of a function giving the maximum flow based on a cap on the capacity of edges of the network and an iterative process for achieving successively better approximations. It has a time complexity of $O(m|V||E|\log_{|E|/(|V|log|V|)}|V| + |E|^2)$ [5], which is better than the complexity of Kishimoto's algorithm, $O(m|V|^3/\log|V|)$ [13].

Another area in which this procedure could be improved is in picking the largest k such that the network can provide continuous k-barrier coverage. A network is able to provide continuous k-barrier coverage if there is a k-route flow through the network of value ≥ 1. A good approach is to decide on the maximal k by conducting a binary search of the integers $\{0, ..., \lfloor F \rfloor\}$ where F is the maximum s-t flow through the network until it finds the optimum flow. This would increase the complexity of the algorithm by a factor of $\log(F)$.

VI. Conclusion

In this paper we have incorporated energy harvesting into the barrier coverage problem and found optimal solutions to the problems of maintaining continuous barrier coverage in energy harvesting sensor networks. Using these techniques we are able to extend the lifetime of a sensor network indefinitely and propose a new metric for measuring the quality of service of a sensor network, the maximum continuous barrier coverage. We have derived the upper bounds on this metric and demonstrated that those bounds could be achieved in polynomial time with algorithms that we have designed. Further, our methods demonstrate that the problem of limited battery capacity can be avoided.

Acknowledgment

This research was supported in part by the Department of Defense in partnership with the NSF REU grant 1156712.

References

[1] C.C. Aggarwal and J.B. Orlin. On multiroute maximum flows in networks. *Networks*, 39(1):43–52, 2001.

[2] S.W. Arms, C.P. Townsend, D.L. Churchill, J.H. Galbreath, and S.W. Mundell. Power management for energy harvesting wireless sensors. In *Smart Strucutres and materials 2005: Smart Electronics, MEMS, BioMEMS, and Nanotechnology*, pages 267–275, 2005.

[3] R. Diestel. *Graph Theory*. Springer-Verlag, 2010.

[4] I. Dietrich and F. Dressler. On the lifetime of wireless sensor networks. *ACM Transactions on Sensor Networks*, 5(1), 2009.

[5] D. Du and R. Chandrasekaran. The multiroute maximum flow problem revisited. *Networks*, 47(2):81–92, 2006.

[6] J.M. Gilbert and F. Balouchi. Comparison of energy harvesting systems for wireless sensor networks. *International Journal of Automation and Computing*, 5(4):334–347, 2008.

[7] J. He and H. Shi. Constructing sensor barriers with minimum cost in wireless sensor networks. *Journal of Parallel Distributed Computing*, 72(12):1654–1663, 2012.

[8] X. Jiang, J. Polastre, and D. Culler. Perpetually environmentally powered sensor networks. In *Proceedings of 4th International Symposium on Information Processing in Sensor Networks (IPSN 2005)*, pages 463–468, 2005.

[9] A. Kansal and M. Srivastava. An environmental energy harvesting framework for sensor networks. In *Proceedings of 2003 International Symposium on Low Power Electronics and Design (ISLPED'03)*, pages 481–486, 2003.

[10] W. Kishimoto. A method for obtaining the maximum multiroute flows in a network. *Networks*, 27(4):279–291, 1996.

[11] W. Kishimoto and M. Takeuchi. On m-route flows in a network. In *Communications on the Move: Singapore ICCS/ISITA'92*, pages 1386–1390, 1992.

[12] S. Kumar, T.H. Lai, and A. Arora. Barrier coverage with wireless sensors. In *Proceedings of 11th International Conference on Mobile Computing and Networking (MobiCom 2005)*, pages 284–298, 2005.

[13] S. Kumar, T.H. Lai, M.E. Posner, and P. Sinha. Maximizing the lifetime of a barrier of wireless sensors. *IEEE Transactions on Mobile Computing (TMC)*, 9(8):1161–1172, 2010.

[14] C. Mathúna, T. O'Donnell, R.V. Martinez-Catala, J. Rohan, and B. O'Flynn. Energy scavenging for long-term deployable wirelss sensor networks. *Talanta*, 75(3):613–623, 2008.

[15] V. Raghunathan, S. Ganeriwal, and M.B. Srivastava. Emerging techniques for long lived wireless sensor networks. *IEEE Communications Magazine*, 44(4):108–114, 2006.

[16] M.K. Stojcev, M.R. Kosanovic, and L.R. Golubovic. Power management and energy harvesting techniques for wireless sensor nodes. In *Proceedings of 9th International Conference on Telecommunication in Modern Satellite, Cable, and Broadcasting Services (TELSIKS '09)*, pages 65–72, 2009.

[17] S. Sudevalayam and P. Kulkarni. Energy harvesting sensor nodes: survey and implications. *IEEE Communications Surveys & Tutorials*, 13(3):443–461, 2011.

[18] C.M. Vigorito, D. Ganesan, and A.G. Barto. Adaptive control of duty cycling in energy-harvesting wireless sensor networks. In *Proceedings of 4th IEEE Communications Society Conference on Sensor, Mesh and Ad Hoc Communications and Networks (SECON'07)*, pages 21–30, 2007.

[19] C. Wang, B. Wang, H. Xu, and W. Liu. Energy-efficient barrier coverage in WSNs with adjustable sensing ranges. In *Proceedings of IEEE 75th Vehicular Technology Conference (VTC Spring)*, pages 1–5, 2012.

[20] L. Xie, Y. Shi, Y.T. Hou, and W. Lou. Wireless power transfer and applications to sensor networks. *IEEE Wireless Communications Magazine*, 2013.