Extending the Lifetime of Barrier Coverage by Adding Sensors to a Bottleneck Region

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***Abstract*—One important application of wireless sensor networks (WSNs) is intruder detection for protection of a region of interest, and *k*-barrier coverage is a good way to achieve the necessary level of security using WSNs. One of the biggest problems with WSNs is their limited lifetime, which is directly related to the deployment and maintenance costs, and many approaches have been proposed to improve network lifetime. In this paper, a new approach to extending network lifetime through strategic deployment of additional sensors is presented. We propose two strategies for deployment of additional sensors: random and targeted. We also analyze their performance through extensive simulation. The key concept for the targeted additional sensor deployment is minimal bottleneck, which limits the network lifetime. We develop an algorithm for finding a bottleneck for 1-barrier coverage. We apply the strategies for deployment of additional sensors to a WSN initially randomly deployed. The targeted additional sensor deployment strategy extends the network lifetime 24% on average with only additional 1.4% of the initial randomly deployed sensors added to the WSN.**

I. Introduction

Applications of WSNs include maintaining environmental homeostasis, controlling industrial processes, monitoring health conditions, and detecting abnormalities in the region of interest. Establishment of surveillance and security to protect the region of interest is one of the most studied applications of WSNs. This type of applications can be found in homeland security, protection of restricted facilities, battlefield surveillance, and intruder detection.

The required level of security is achieved with a WSN by deploying hundreds or even thousands of sensors. The deployed sensors establish surveillance over the given area within their sensing ranges. Although area coverage where every location in the region of interest is monitored by a sensor can be used to protect the region and detect the intruders, barrier coverage is more suitable, because barrier coverage requires much fewer sensors to achieve the necessary level of security [1]. A k-barrier coverage is formed with k disjoint sets of sensors. Each barrier is a path of sensors from the source to the sink, and the sensing ranges of neighboring sensors overlap. These barriers are also referred to as strong barriers [2]. Barriers must be strong to guarantee detection of intruders. A k-barrier coverage maintains k number of barriers active at any time to ensure that an intruder will be detected by at least k sensors. In this paper, we focus on the 1-barrier coverage.

WSNs used to establish barrier coverage face some limitations due to the inherent nature of wireless sensors. Computational capability, communication range, sensing range, limited battery lifetime, and limited memory space are all major issues bound to wireless sensors and thus, WSNs [3]. Due to the limited resources of WSNs, energy expenditure, number of sensors, method of deployment, and scheduling must all be considered to achieve the required performance, such as the lifetime and the number of disjoint barriers. Initial deployment strategies [4], sleep-wake-up scheduling [5], and localized protocols [1] have been studied previously to achieve the most optimal performance given limited resources.

Increasing the lifetime of a WSN for k-barrier coverage is one of the most active fields of research, because lifetime is directly related to deployment and maintenance costs. With longer lifetime of the WSN, the number of deployed sensors can be smaller, and the re-deployment of the WSN can be less frequent, while still maintaining the same performance.

To the best of our knowledge, there has not been any study on extending the lifetime of k-barrier coverage through deployment of additional sensors. In this paper, we first define bottlenecks and study their properties related to lifetime. We focus on the 1-barrier coverage in developing the algorithms to locate a minimal bottleneck and to compute the network lifetime. We then investigate two strategies for deployment of additional sensors to extend the lifetime of WSN. The two strategies are referred to as random and targeted strategies, and the targeted additional sensor deployment uses the bottleneck properties to extend the lifetime of WSN. We compare the two strategies through extensive simulation.

The rest of the paper is organized as follows. In Section II, the related work is outlined and discussed. The network model and the assumptions about the WSN are listed and explained in Section III. Section IV describes in detail the two strategies for deployment of additional sensors and defines the related terms. The algorithms, the simulation, and the related parameters are presented in Section V. The simulation results are analyzed and compared for the two additional sensor deployment strategies in Section VI. In Section VII, the paper is concluded, and the future work is discussed.

II. Related Work

Various deployment strategies for area coverage have been well studied [6]. Xu et al. study different deployment strategies, which are connectivity-oriented, lifetime-oriented, and hybrid, without a device homogeneity assumption and evaluate the performance with large-scale WSN simulation. The difference between the optimal deterministic deployment and the random deployment is studied with consideration of placement errors for the case of 1-full area coverage [7], and three popular deployment strategies are compared while varying the network parameters.

There have been several studies on deployment strategies for the *k*-barrier coverage problem [4], [8]. Yang and Qiao propose a multi-round sensor deployment and evaluate the performance through analytical study and simulation. They provide not only a solution to the problem of guaranteed barrier coverage using two-round deployment but also two practical solutions to the realistic situation with limited knowledge about the deviation of the sensors' actual positions with respect to their intended positions. Eftekhari et al. extend the work of Yang and Qiao by introducing two multi-round deployment strategies, one partial and one complete, and computing the probability of *k*-barrier coverage as a function of network parameters [8]. Eftekhari et al. also analyze the total expected cost of two deployment methods and validate the analysis with simulation results.

A localized protocol that guarantees local barrier coverage and a sleep-wake-up algorithm that achieves near optimal network lifetime are introduced in [1]. Kumar et al. further investigate sleep-wake-up scheduling for both the homogeneous and heterogeneous lifetime cases and propose a centralized optimal sleep-wake-up algorithm that maximizes the network lifetime given node-disjoint paths [5].

Li et al. propose an incremental sensor deployment algorithm minimizing the total communication power consumption and validate the algorithm performance through simulation [9]. Du et al. introduce a scheduling algorithm, which combines the concept of redeployment and the sleep-wake-up schedule, for *k*-barrier coverage using mobile sensors [10]. They focus not only on the probability of *k*-barrier coverage, but also on the extension of lifetime.

Although various deployment strategies for WSNs have been studied [4], [6]–[8], this paper differs in the coverage problem, deployment type, and deployment goals. Unlike the work of Xu et al. [6] and Balister and Kumar [7], which are for the area coverage problem, this paper is specific to the barrier coverage problem. Although the work of Yang and Qiao [4] and Eftekhari et al. [8] are for *k*-barrier coverage, this paper is not focused on initial deployment strategies but on strategies for deployment of additional sensors. Also, their work focuses on establishing of *k*-barriers and reducing the number of deployed sensors, respectively, while this paper focuses on extending the lifetime of the network. This paper is more closely related the work of Du et al. [10], both focusing on increasing the lifetime of the network for the *k*-barrier coverage problem utilizing the concept of strategic deployment. However, they have different approaches and assumptions and thus, face different challenges. The concepts of mobile sensors, scheduling, and redeployment are used in [10], while the concepts of immobile sensors, bottlenecks, and additional sensor deployment are used in this paper.

III. Network Model

The network model used in this paper is similar to the network model used in [3]. The region where the sensors are deployed is a thin belt region with a virtual source at one end and a virtual sink at the other end. Since the special initial deployment strategy might not be practical or possible depending on the availability of resources and the application, the sensors are initially randomly deployed in the thin belt region. However, the additional deployment strategies presented in this paper can still be applied to WSNs with a special initial deployment strategy. The deployed sensors are immobile, and thus, the sensors cannot relocate themselves to improve the connectivity, the coverage, or the lifetime of the WSN.

Unlike the network model used in [3], the sensors are fully localized. This is a restrictive assumption on the network model, but it is not an unreasonable assumption, since localization information is required in many applications and localization algorithms have been extensively studied [11]. Also, the algorithms for computing the lifetime of WSN and finding the bottleneck are all centralized algorithms.

The deployed sensors are homogeneous. Thus, they all have the same communication range and the same sensing range. Furthermore, the communication range is twice the sensing range. This is to ensure that coverage implies connectivity [12]. A path from the source to the sink is found by connecting the neighbors sequentially. Since the two sensors are considered to be *neighbors* if their sensing ranges overlap or are tangent, neighboring sensors in a barrier can alway communicate.

The energy model adopted in this paper is similar to that adopted in [13]. The sensing energy consumption rate is proportional to the square of the sensing range, and the initial energy level is different in each sensor. Since the sensing ranges of the sensors are the same, the sensing energy consumption rates of all the sensors are the same. However, the lifetimes, which depend both on the initial energy level and the sensing energy consumption rate, of the sensors may be different. The homogeneity of the sensing range does not cause loss of generality in terms of determining the sensors' lifetimes, because the lifetimes are already heterogeneous due to the heterogeneous initial energy levels. Heterogeneous sensing range will only increase the variance of the sensors' lifetimes, which can also be achieved by increasing the variance of the initial energy level distribution.

IV. Additional Sensor Deployment

The two strategies for deployment of additional sensors are presented in this section. We first define a few terms which are

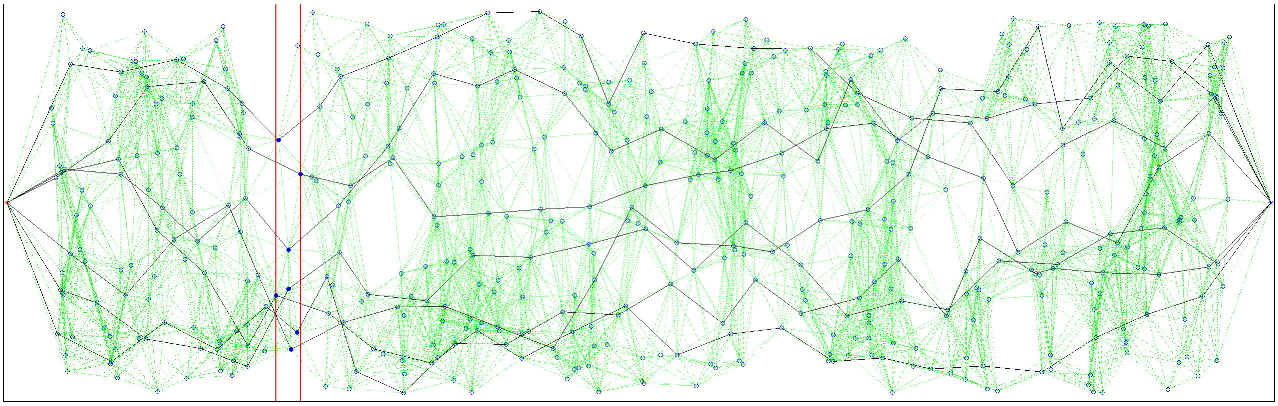

Fig. 1. Sample WSN topology with a bottleneck region

critical in understanding the two additional sensor deployment strategies.

A. Definitions

The definitions below are specific only to the k-barrier coverage.

- A given WSN *stops functioning* when there are less than k disjoint paths from the source to the sink.
- The *network lifetime* is the total duration of time from the start to the time the network stops functioning.
- A *bottleneck* is a set of sensor nodes which make k disjoint paths no longer possible in the network if all of them are dead.
- The *total lifetime of a bottleneck* is the sum of the lifetimes of the nodes belonging to the given bottleneck.
- A *bottleneck region* is a rectangular region from the leftmost bottleneck node to the rightmost bottleneck node.

Figure 1 shows a sample WSN topology in the thin belt region. The circles (blue) are the deployed sensors, and the dotted lines (green) connect the neighboring nodes whose sensing ranges overlap. The virtual source is located at the left end, and the virtual sink is located at the right end. The solid lines (black) connecting the source and the sink are the disjoint strong barriers. The set of the filled circles is a minimal bottleneck of this particular WSN, and the region between the red lines is a minimal bottleneck region.

In any WSN, there exist bottlenecks throughout the network, and a minimal bottleneck puts an upper bound not only on the network lifetime, but also on the maximum number of disjoint barriers. Therefore, the region where a minimal bottleneck exists is a weak area, and strengthening that region will greatly improve the network lifetime.

B. Strategies for Deployment of Additional Sensors

Our approach to extending the lifetime of a WSN for k-barrier coverage is to deploy additional sensors to a minimal bottleneck region. We investigate two types of additional sensor deployment strategies and compare their performance. One type of additional sensor deployment strategies is *random additional sensor deployment*, which extends the network lifetime by deploying the additional sensors anywhere in the entire thin belt region randomly. The other type of additional sensor deployment strategy is *targeted additional sensor deployment*, which extends the network lifetime by deploying the additional sensors anywhere in a minimal bottleneck region randomly. The only difference between the two additional sensor deployment strategies is the region of deployment.

V. Simulation

The algorithms are implemented in MATLAB. The simulation is done in MATLAB for the effect of additional sensors deployed on the lifetime. As the additional sensor deployment percentage, x, is varied, the network lifetimes with only the initial deployment, with the random additional sensor deployment after the initial deployment, and with the targeted additional sensor deployment after the initial deployment are computed using a maximum flow algorithm. Since several other deployment factors, such as initial sensor positions, additional sensor positions, and initial energy levels, affect the performance, the simulation results vary from one deployment to another. Therefore, 2000 rounds of simulation are done for each value of x, which ranges from 0.2% to 10% with an increment of 0.2%, to reduce the effect due to the randomness of other deployment factors.

A. Simulation Parameters

In the simulation, the initial deployment count is 500 sensors, and all of the initially deployed sensors are randomly placed in a 5×30 thin belt region. Since it is desirable for any two sensors with overlapping sensing ranges to be able to communicate, the communication range is twice the sensing range. Thus, the communication range r_C is set to 1.5, and the sensing range r_S is set to 0.75 for all the sensors. Adopting the sensing energy model of Wang and Medidi [13], the sensing energy consumption rate is set to $0.01 \cdot r_S^2$, and the initial energy level distribution for the sensors is a uniform distribution from 40 to 50 inclusive. Only the energy consumption related to sensing is considered in calculating the lifetimes of sensors. For simplicity, only the 1-barrier coverage is simulated.

B. Barrier Coverage Algorithm

To compute the network lifetime, we apply the Ford-Fulkerson algorithm, a maximum flow algorithm [14]. For the Ford-Fulkerson algorithm to be applicable to finding disjoint barriers, every sensor must be transformed, and the edge capacity must be assigned in a special way. The way in which the sensors transform is very similar to that adopted in [3]. Each sensor v must be split into two virtual subnodes, v_a and v_b, where v_a and v_b can instantly exchange information. The virtual subnode v_a can only receive information from its neighbors' subnode b and the virtual subnode v_b can only transmit information to its neighbors' subnode a. The splitting of a sensor into two virtual nodes is necessary to avoid using a sensor longer than its lifetime.

The edge capacities are related to the sensors' lifetimes. The lifetime L_v of a sensor v is

$$L_v = \frac{E_v}{0.01 \cdot r_S^2},$$

where E_v is the initial energy level of sensor v. The inter-node capacity is the minimum of the lifetimes of the two connected sensors, and the intra-node capacity is the lifetime of a given node. The sensor transformation and the edge capacities are summarized in Fig. 2 below.

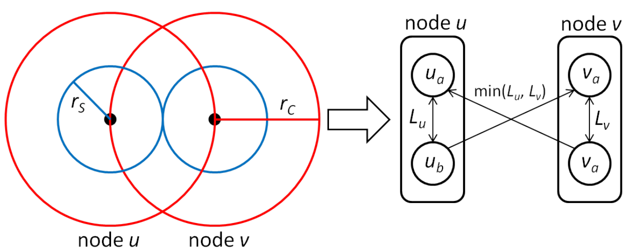

Fig. 2. Sensor transformation and edge capacity assignments

The Ford-Fulkerson algorithm is different from the minimum-cost maximum-flow algorithms which are usually applied to k-barrier coverage to find disjoint barriers. Unlike the minimum-cost maximum-flow algorithms, the Ford-Fulkerson algorithm does not necessarily find disjoint paths. The minimum-cost maximum-flow algorithm might have to be iterated few times until the network is fully utilized to compute the network lifetime. However, the network lifetime can be computed with one application of the Ford-Fulkerson algorithm.

For 1-barrier coverage, the network lifetime is the sum of all the flows from the source to the sink. Since the Ford-Fulkerson algorithm does not necessarily find disjoint paths, a special scheduling algorithm must be implemented to compute the network lifetime for k-barrier coverage with any integer $k > 1$, which may be developed in the future.

C. Minimal Bottleneck Finding Algorithm

For 1-barrier coverage, the definition of bottleneck can be interpreted as a set of nodes which make a path from the source to the sink no longer possible when they are all dead. The bottleneck nodes are a subset of the weakest nodes in each path with the least lifetime, that is, the least remaining energy. Therefore, the WSN with the weakest nodes considered dead is divided into two regions, where one region is reachable from the source and the other region is not reachable from the source. After a given WSN stops functioning, the sensors can be categorized into three types, which are dead, reachable, or unreachable. The hop count from the source can be found for each node. A reachable node has a finite hop count, while an unreachable has an infinite hop count. A minimal bottleneck is a subset of the dead sensor nodes. A dead node is in the minimal bottleneck if it is in a chain of dead nodes with one end of the chair connected to a reachable node and the other connected to an unreachable node.

An algorithm for finding a minimal bottleneck for 1-barrier coverage is devised using the concepts of hop count, reachable node, and unreachable node, as shown in Algorithm 1.

Since the sensors are localized, it is possible to find the x coordinates of the leftmost and the rightmost minimal bottleneck nodes. Therefore, it is possible to determine the minimal bottleneck region for the targeted additional sensor deployment once the minimal bottleneck is found.

D. Simulation Procedure

The 500 sensors are initially deployed randomly on the given field, and the network lifetime is computed using the Ford-Fulkerson algorithm. The bottleneck and the bottleneck region are found using the smallest bottleneck algorithm. On the same initial WSN, additional $x\%$ of 500 sensors are deployed using the two additional sensor deployment strategies separately creating two strengthened WSNs. For each strengthened WSN, the lifetime is computed using the Ford-Fulkerson algorithm. The three lifetimes corresponding to the initial and two strengthened WSNs are compared.

VI. Results

The percentage of improvement in lifetime by the two strategies for additional sensor deployment is computed for each value of x, and the lifetime improvement is shown in Fig. 3.

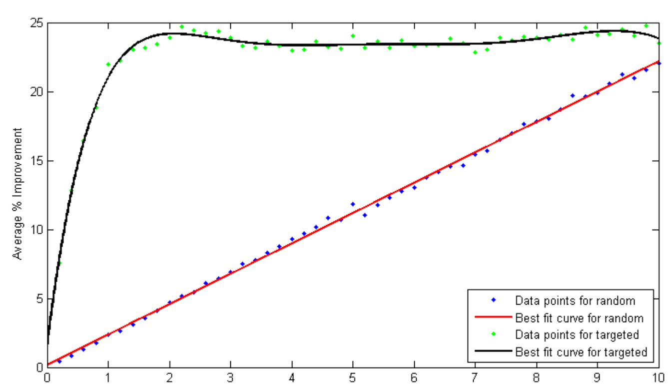

Fig. 3. Percentage of improvment in lifetime

Algorithm 1 Minimal Bottleneck Finding Algorithm

Input: List of all nodes L, neighbor list N, & energy level e
Output: List of minimal bottleneck nodes B

Compute hop count h_v, $\forall\ v \in L$, starting from the source
for $v \in L$ **do**
  if $e_v = 0$ **then**
    Append v to the list of dead nodes D
  else
    if $h_v < \infty$ **then**
      Append v to the list of reachable R
    else
      Append v to the list of unreachable U
    end if
  end if
end for
for $v \in D$ **do**
  if $(N_v \cap R \neq \varnothing) \wedge (N_v \cap U \neq \varnothing)$ **then**
    Append v to B
  else if $(N_v \cap R \neq \varnothing) \oplus (N_v \cap U \neq \varnothing)$ **then**
    for $w \in (D \cap N_v)$ **do**
      if $((N_v \cap R \neq \varnothing) \wedge (N_w \cap R = \varnothing) \wedge (N_w \cap U \neq \varnothing)) \vee ((N_v \cap U \neq \varnothing) \wedge (N_w \cap R \neq \varnothing) \wedge (N_w \cap U = \varnothing))$ **then**
        Append v to B
      end if
    end for
  else
    Append v to the list of intermediate nodes I
  end if
end for
Initialize temporary list $T = \varnothing$
Initialize potential bottleneck list $P = \varnothing$
for $v \in I$ **do**
  if $v \notin P$ **then**
    Append v to T
    while $(I \cap N_u \setminus T) \neq \varnothing,\ \forall u \in T$ **do**
      Append all nodes in $I \cap N_u \setminus T,\ \forall u \in T$ to T
    end while
    if $B \cap N_u = \varnothing,\ \forall\ u \in T$ **then**
      Append u and N_u's nodes, $\forall\ u \in T$, to P
    end if
  end if
  Set $T = \varnothing$
end for
Append all P's nodes to B

The average percentage improvement in lifetime with the random additional sensor deployment increases linearly as the value of x increases. Since the minimal bottleneck limits the network lifetime, the additional sensors that do not land in the minimal bottleneck region will not extend the network lifetime. As the value of x increases, sensors that are randomly deployed anywhere in the entire field will also increasingly land in the minimal bottleneck region, thus increasing the network lifetime. This is why the average percentage improvement in lifetime for the random additional sensor deployment is a linear function of x.

The average percentage improvement in lifetime with the targeted additional sensor deployment increases steeply until about $x = 1.4\%$, and the curve flattens afterwards, which is indicative of diminishing returns of additional sensors. This diminishing return effect will be further discussed later in this section. It is interesting to note that there exists an optimal percentage of extra sensors for the targeted additional sensor deployment.

If the trend in the curve for the random additional sensor deployment continues, the two curves will intersect at about $x = 11.0\%$. This implies that the targeted additional sensor deployment can achieve the same level of the percentage lifetime extension with only about 13% of the number of sensors required by the random additional sensor deployment. However, the targeted additional sensor deployment suffers from the diminishing return effect and cannot extend the network lifetime more than a certain percentage, while the random additional deployment does not have such a limitation.

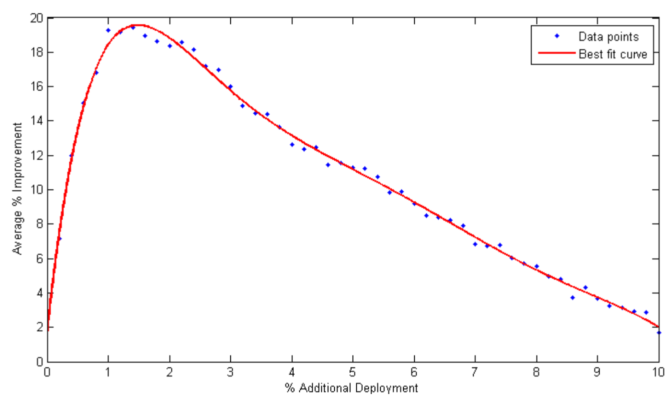

Fig. 4. Percentage lifetime extension of targeted relative to random

Figure 4 shows a curve of the average percentage difference between the network lifetime improvement by the targeted additional sensor deployment and the network lifetime improvement by the random additional sensor deployment. The optimal value for the additional sensor deployment percentage for the targeted additional sensor deployment is $x = 1.4\%$, as shown in the figure.

A. Effects of Additional Sensor Deployments

The random additional sensor deployment has the effect of increasing the total lifetime of every bottleneck, but not necessarily the minimal bottleneck found by the minimal bottleneck finding algorithm. Therefore, the random additional sensor deployment strategy may not necessarily increase the network lifetime at low values of x, as the additional sensors are distributed randomly to all the bottlenecks. This is why the average percentage improvement in lifetime is very small at low values of x.

As the targeted additional sensor deployment only increases the total lifetime of the minimal bottleneck, the lifetime improvement is noticeable at any value of x. At high values of x, however, the targeted additional sensor deployment strategy suffers from the diminishing return effect. The diminishing return effect occurs, because if the total lifetime of the minimal bottleneck is sufficiently increased, the next minimal bottleneck will become a new minimal bottleneck limiting the network lifetime, and thus, keeping increasing the total lifetime of the original minimal bottleneck will no longer increase the network lifetime, which is now dependent on the new minimal bottleneck. At very high values of x, the random additional sensor deployment is better than the targeted additional sensor deployment, because random additional sensor deployment is more likely to increase the total lifetime of not only one minimal bottleneck, but also all the minimal bottlenecks in the network, while the targeted additional sensor deployment only increases the total lifetime of one minimal bottleneck.

VII. Conclusion

This paper focuses on extending the lifetime of WSN for 1-barrier coverage. We have developed a new approach to improving network lifetime which is directly related to the deployment and maintenance costs. The additional sensor deployment to the initial WSN is a concept of repairing the WSN's weak areas which are bottleneck regions. A minimal bottleneck and the minimal bottleneck region limits the network lifetime to the total lifetime of the minimal bottleneck.

The two strategies for additional sensor deployment, which are random and targeted, are simulated on the same initial WSN. As the percentage of additional sensor deployment x is varied, the corresponding average percentage improvements in lifetime of the two strategies are computed and analyzed. Although the targeted additional sensor deployment suffers from the diminishing return effect at high values of x, the targeted additional deployment can achieve about 24% improvement in the network lifetime at low values of x, and the optimal value of x is 1.4%. Although the random additional senor deployment has very small average percentage improvement in lifetime at low values of x, it can extend the network lifetime without the diminishing return effect as the value of x increases. These results are due to the fact that the random additional sensor deployment has an effect of increasing the total lifetime of all the bottlenecks throughout the WSN, while the targeted additional sensor deployment only increases the total lifetime of one minimal bottleneck. Because of the nature of the targeted additional sensor deployment, the optimal value of the additional sensor deployment percentage exists and depends on the WSN parameters, such as the initial deployment count and the initial deployment region. It can be concluded that the main advantage of the targeted additional deployment is that it can extend the network lifetime much more efficiently using much fewer sensors compared to the random additional deployment.

This research may be extended to the k-barrier coverage for any positive integer k. In such case, the network lifetime algorithm and the minimal bottleneck finding algorithm must be revised, since those algorithms presented in this research use the unique property of 1-barrier coverage. This research may also be extended by removing some of the restrictive assumptions, such as homogeneous sensing and communication ranges. For more realistic simulations, the energy consumption related to other sources, the localization error, and the deployment error may be considered.

It is also possible to investigate the effects of the repeated additional sensor deployments using the two deployment strategies presented in this research. It will be interesting to compare the percentage improvements in lifetime after deploying the optimal percentage of additional deployment in each of N minimal bottlenecks in a given WSN and after deploying the same total number of sensors randomly in the entire field. The effects of additional sensor deployment on other performance metrics such as the maximum number of disjoint barriers can also be studied as future work.

Acknowledgment

This research is funded by National Science Foundation REU award #1156712 that is co-funded by the Department of Defense.

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